Importance of Intrinsic-Q in Microring-Based Optical Filters and Dispersion-Compensation Devices

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***Abstract*—We investigate the impact of the intrinsic-Q of the resonant poles on the performance of multiring-based optical filters and dispersion-compensating devices. We highlight the role of quality factor by defining figures-of-merit for some specific filter configurations.**

***Index Terms*—Dispersive media, optical filters, optical microresonators.**

I. Introduction

OPTICAL microring resonators (OMRs) are versatile elements for designing integrated photonic circuits, and can be used as building blocks for many optical signal processing devices and systems. It has been shown that OMRs can serve as poles in multipole add–drop filters [1]–[3] and multistage dispersion compensators [4]–[6]. Although significant effort has been dedicated toward design and fabrication of these devices, their performance has not been clearly characterized in terms of the intrinsic quality factor of the constituent microring resonators (Q_0). Note that Q_0 is limited by intrinsic loss mechanisms (i.e., material loss and scattering losses due to surface roughness) and is a characteristic of different microresonator fabrication technologies and material systems (polymer, semiconductor, hydex, etc.). Hence, Q_0-characterization simplifies the comparison and evaluation of the performance of optical multiring filters built based on different platforms and technologies. Here we present an alternative perspective on the theory of multiring filters that highlights the role of intrinsic quality factor in over-all behavior of these devices. We show that the interplay between coupling and loss makes the attainment of high intrinsic quality factor an essential requirement for designing multiring bandpass filters and all-pass filters (dispersion compensators).

II. Microring-Based Bandpass Filters

Fig. 1(a) shows the most common configuration used for designing microring-based multipole bandpass filters. The behavior of the multipole filters is well understood based on their building blocks, i.e., the single-pole (single-ring) add–drop filter [Fig. 1(b)]. The single-pole filter has a Lorentzian power transfer function [Fig. 1(c)] with a bandwidth defined by $2\delta = \nu_0/Q_{\mathrm{tot}}$, where $Q_{\mathrm{tot}}^{-1} = Q_b^{-1} + Q_d^{-1} + Q_0^{-1}$, Q_b and Q_d are external quality factors associated with coupling to bus and drop waveguides ($Q_{\mathrm{ext}} \sim 2\pi^2 R n_e/\lambda_0\kappa^2$; R: microring radius; n_e: effective refractive index of the optical mode; κ: coupling factor for the electric field [1]). The total power transfer ($L = P_{\max}/P_{\mathrm{in}}$) is a function of Q_0 and the coupling factors (κ). Note that the magnitude of the coupling factors control the distribution of the input power among the reflected, dropped, and throughput powers.

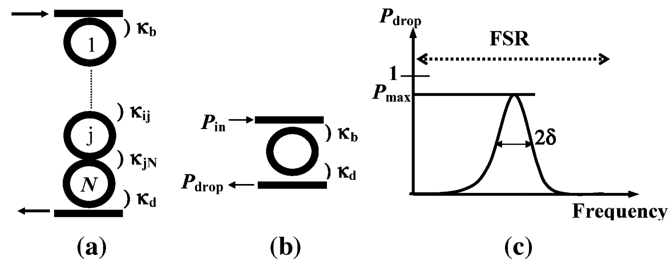

Fig. 1. (a) Schematic diagram of a multiring filter. (b) Single-pole (single-ring) filter. (c) Dropped power spectrum for a single-pole filter.

The minimum insertion loss (maximum power transfer from input to the drop channel) occurs for an asymmetric coupling configuration in which the input coupling is set at the critical coupling condition ($Q_b^{-1} = Q_d^{-1} + Q_0^{-1}$). Note that we assume all coupling junctions are lossless. It has been shown that the power transfer for a critically coupled single-ring filter is equal to $L_{\max} = 1 - 2Q_{\mathrm{tot}}/Q_0$ [7]. However, here we consider the symmetric coupling configuration (i.e., $Q_b = Q_d$ or $\kappa_b = \kappa_d$, therefore, critical coupling is not perfectly attained) that simplifies insertion loss calculations for higher order filters [1]. For a single-pole symmetric coupler the power transfer can be written as $L_s = (1 - Q_{\mathrm{tot}}/Q_0)^2$. Note that for relatively small values of Q_{tot}/Q_0, the difference between the symmetric coupler and critically coupled coupler is negligible. If we define $Q_{\mathrm{tot}}^{(1)}$ as $(0.5 \times Q_b^{-1} + Q_0^{-1})^{-1}$, or basically the Q_{tot} for a symmetric single-pole element, L_s for higher order filters can be derived as a function of $Q_0/Q_{\mathrm{tot}}^{(1)}$. Regardless of the order and the shape of the filter, L_s always increases for larger values of $Q_0/Q_{\mathrm{tot}}^{(1)}$. Practically, this means that the insertion loss can be reduced by either increasing the operational bandwidth (through waveguide loading) beyond the bandwidth already set by the resonator itself, or increasing the intrinsic-Q.

In the multiring configuration the spectral response of the transfer function is determined by the relation among the coupling factors. Photonic counterparts of the well-known RF filter

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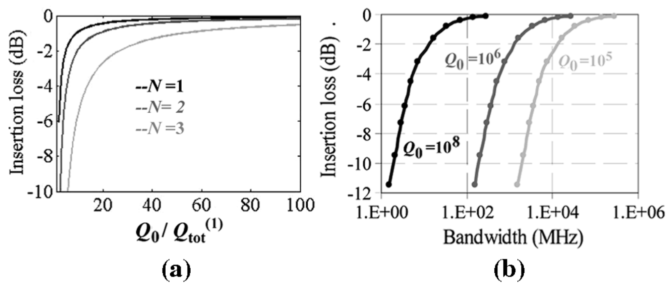

Fig. 2. (a) Insertion loss plotted against $Q_0/Q_{\rm tot}^{(1)}$ for three filter orders with Butterworth transfer function. (b) Insertion loss plotted against 3-dB bandwidth for three values of Q_0 in a third-order Butterworth filter. The intrinsic-Qs are chosen based on available microring technologies: semiconductor and polymers (10^5) [8], [9], Hydex (10^6) [2], and silica microtoroids (10^8) [7].

functions such as Chebyshev and Butterworth (maximally flat) can be realized by carefully tuning the mutual coupling factors (κ_{ij}) with respect to κ_b and κ_d [1]. Here we consider a symmetric $(\kappa_d = \kappa_b)$ multiring coupler with Butterworth (maximally flat) response. For any filter order (N), this response can be achieved by setting the proper relation between κ_d and $\kappa_{ij}s$. Fig. 2(a) shows the insertion loss plotted against $Q_0/Q_{\rm tot}^{(1)}$ for $N = 1, 2$, and 3. As evident from the graph a large value of $Q_0/Q_{\rm tot}^{(1)}$ is even more important as the filter order increases. Insertion loss scales as $K \times (Q_0/Q_{\rm tot}^{(1)})^{-0.85}$, where $K = -6$ dB, -14 dB, -32 dB, for $N = 1, 2$, and 3, respectively. Note that $Q_{\rm tot}^{(1)}$ is inversely proportional to κ_d and basically carries the signature of all coupling strengths for a specific filter function.

In order to illustrate the effect of insertion loss compensation by over-coupling (decreasing $Q_{\rm tot}^{(1)}$), the insertion loss of a third-order Butterworth filter is plotted against its 3-dB bandwidth for three values of Q_0 [Fig. 2(b)]. As shown in the plot, a low insertion loss is accompanied with a bandwidth penalty and clearly a lower bandwidth requires larger intrinsic-Qs. An equally important figure-of-merit in a bandpass filter is the roleoff slope that represents the strength of the out-of-band signal rejection. For any filter shape, this slope is proportional to the order of the filter N^x, where x is the determined by the specific filter function). One of the challenging tasks for photonic filter design is building a narrow bandwidth box-like (large slope) filter with low insertion loss. These three characteristics are essential in many optical communication and RF-photonic signal processing systems where narrowband, low loss, and high rejection of closely spaced frequency components is required [10], [11]. Although the exact Q_0-dependence of insertion loss and roleoff varies in different filter designs, it is useful to define a figure-of-merit that highlights the role of quality factor in the overall performance of a filter. Knowing that insertion loss approximately scales as N/Q_0, bandwidth scales as $1/NQ_0$, and the slope scales as NQ_0, the figure-of-merit can be written as

$$\frac{\textbf{slope}}{\textbf{Ins.Loss} \times \textbf{BW}} \propto \frac{\boldsymbol{NQ_0}}{(\boldsymbol{N/Q_0})(\boldsymbol{1/Q_0N})} = \boldsymbol{NQ_0^3}.$$

This figure-of-merit illustrates the extreme importance of high intrinsic-Q in the design of box-like multipole bandpass filters.

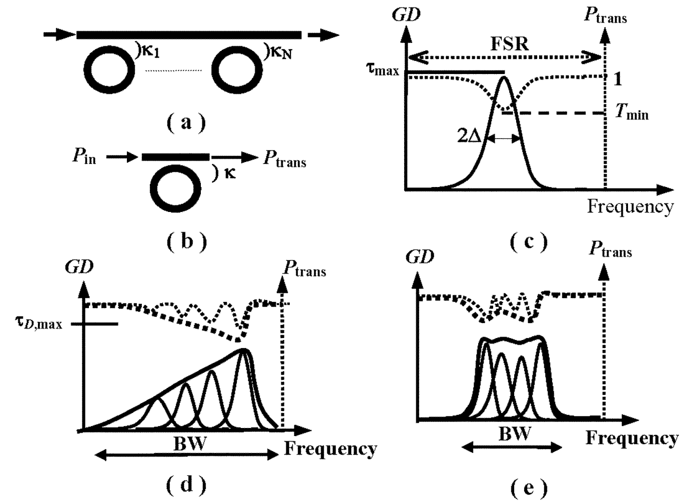

Fig. 3. (a) Multiring APF architecture. (b) Single-pole microring APF. (c) GD and insertion loss spectrum of a single-pole APF. (d) Four-pole APF with constant dispersion. (e) Four-pole APF with constant delay over a passband.

III. Microring-Based Dispersion-Compensating Stages

It is well known that microrings can be used to build all-pass optical filters (APFs) [4], [5]. Ideally, APFs have a constant amplitude response and a phase response that can be made arbitrarily close to any desired response. It has been shown that photonic all-pass filters can be realized by side coupling a series of OMRs to a waveguide [4], [5]. Fig. 3(a) shows the schematic diagram of a multiring optical APF. The group delay (GD) spectrum of a single-pole ring filter [Fig. 3(b)] has a Lorentzian shape [Fig. 3(c)] and its maximum GD $(\tau_{\max})$ is proportional to $Q_{\rm tot}(Q_{\rm tot}^{-1} = Q_b^{-1} + Q_0^{-1})$ [12]. Note that due to absence of the drop port, in this configuration, $Q_{\rm ext} = Q_b$. The power transfer $(P_{\rm trans}/P_{\rm in})$ spectrum of this filter can be characterized by an inverted Lorentzian with a minimum transmission of $T_{\min}$ [Fig. 3(c)].

It is desirable to maintain a unity amplitude response over the bandwidth of an APF. In a ring-resonator-based APF, this requirement translates to operation in the over-coupled regime, where $Q_0/Q_{\rm tot}$ should be large. Since large GD $(\tau_{\max})$ is also desirable and $\tau_{\max} \propto Q_{\rm tot}$, high intrinsic-$Q$ operation becomes a critical aspect of APF filter design (to increase $Q_0/Q_{\rm tot}$ without sacrificing $\tau_{\max}$). By adjusting the coupling factors (κs) and the relative resonant frequency of the constituent microring resonators, the phase response of multiring APFs can be tailored to serve as linear dispersion compensators [Fig. 3(d)] that can be used in wavelength-division-multiplexed communication systems [13], or as optical delays [Fig. 3(e)] that are useful in applications like RF-photonic signal processing and RF-phased arrays [14].

Assuming all rings to have an intrinsic quality factor of Q_0, adjusting the coupling factors (κs) determines the $T_{\min}$ and $\tau_{\max}$ for each pole and, therefore, the overall GD spectrum. There is always an undesired frequency-dependent loss associated with any GD or dispersion response. To generate an amplitude response close to an ideal APF filter, the response $dT_{\min}/d\tau_{\max}$ should be small enough so that the spectral variation of the insertion loss becomes negligible over the

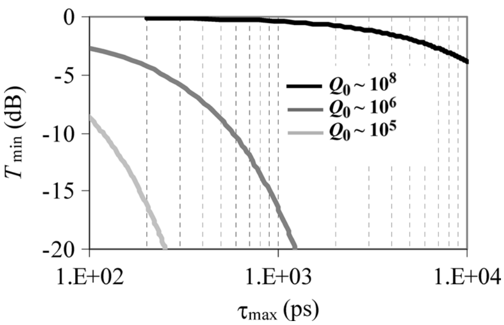

Fig. 4. Minimum transmission plotted against maximum GD for a single-ring APF and for three different values of Q_0.

operational bandwidth. Fig. 4 shows the minimum transmission plotted against maximum GD for a single ring at three values of Q_0. As evident from the plot when Q_0 increases both the maximum insertion loss and the $dT_{\min}/d\tau_{\max}$ decrease significantly at a given $\tau_{\max}$.

Consequently, the intrinsic quality factor of the constituent microring resonators in an optical APF filter play a critical role in achieving large dispersion or delays with a low insertion loss and flat response. Dispersion is defined as $D = d\tau/d\lambda$ (or $d\tau/d\omega$, where τ is the delay and λ and ν are optical wavelength and frequency). For a linear dispersion over a bandwidth [Fig. 3(d)], $D = \tau_{D,\max}/\mathrm{BW}$. So, for a linear dispersion-compensating APF, a figure-of-merit can be defined that combines large dispersion with low loss and a flat response

$$\frac{\textbf{Dispersion}}{\textbf{Ins.Loss} \times \textbf{flatness}} \propto \frac{\frac{Q_0}{N/Q_0}}{(1/Q_0)(1/Q_0)} = \frac{Q_0^4}{N}.$$

In an APF that generates a constant GD within a bandwidth BW [Fig. 3(e)], large dispersion should be replaced with a large delay so, in this case, the figure-of-merit can be written as

$$\frac{\textbf{Group\_delay}}{\textbf{Ins.Loss} \times \textbf{flatness}} \propto \frac{Q_0}{(1/Q_0)(1/Q_0)} = Q_0^3.$$

IV. Conclusion

We have investigated the importance of intrinsic-Q in microring-based optical bandpass and all-pass filters using a simple analysis based on the fundamental properties of these filters. Insertion loss characterization of multiring filters as a function of $Q_0/Q_{\mathrm{tot}}^{(1)}$ is a practical approach for evaluating the performance of these filters regardless of their transfer function and design details. Similarly using the $T_{\min}$ versus $\tau_{\max}$ behavior of the constituent microrings of a compensator enables a quick evaluation of the overall behavior of the filter without involving the details of multipole interactions. Generally speaking, for most applications, the Q_0 of constituent rings in these designs should be made to be as high as possible, even in designs where the operational quality factor (Q_{tot}) is low. High-Q operation is also important from a fabrication point of view since the magnitude of the coupling factor for a given amount of the coupled power to the ring is inversely proportional to Q_0. So monolithic multiring filters with high-Q rings can be designed with larger coupling gaps (smaller coupling factors) that are easier to fabricate. We should note that thermal and nonlinear effects may impose an upper limit on the circulating optical power inside the ring resonators. However, this limit is related to the operational-Q (Q_{tot}) and not the intrinsic-Q (Q_0) of the system. Furthermore, to obtain low insertion loss, Q_{tot} will be much less than Q_0 and hence high Q_0 of the constituent elements of a compound filter does not necessarily impose a power handling limitation on the filter design.

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